

Calculators and mobile phones are not allowed
Answer all of the questions

5 pts) Let $f(x) = e^x - \tan^{-1} x$, $x > 0$.

Prove that f is 1-1.

1) Find the domain and range of f^{-1}

2) Find an equation for the tangent line to the curve $y = f^{-1}(x)$ at the point $(e - \pi/4, 1)$.

6 pts) Find $\frac{dy}{dx}$ if

1) $y = \ln \sqrt{\frac{\sin(e^x)}{\sqrt{x + \sec^{-1} x}}}$

2) $\tan^{-1} \frac{y}{x} = \pi \tan^{-1} x + \sin^{-1}(3x - 2y)$.

(4 pts) Prove the identity

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right), \quad xy \neq 1.$$

(9 pts) Evaluate each integral.

a) $\int \frac{1}{(x^2 + 1) \tan^{-1} x} dx$

b) $\int \frac{\sin x}{1 + e^{\cos x}} dx$

c) $\int \frac{1}{\sqrt{e^{8x} - 9}} dx$ c) $\int \frac{1}{\sqrt{e^{8x} - 9}} dx$